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SCALING LAWS FOR ADIABATIC SHEAR BANDS

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Abstract—Mathematical analysis of a simple, one-dimensional, canonical problem has yielded a number of scaling laws that relate various characteristic features of an adiabatic shear band to the physical properties of the material and the ambient conditions. Specific formulas have been obtained from solutions to linear and non-linear problems coupled with asymptotic representations of the results. Examples are shear band width, most sensitive strain rate and shear band spacing. Other results show the equivalence of initial fluctuations in strength or temperature, and still others show how to scale the defect in formulas for estimating the timing of localization. The paper summarizes previous results and presents some new formulas that show how strain rate sensitivity affects band spacing, number density and morphology as a function of strain rate.

1. INTRODUCTION

Adiabatic shear bands are a major damage mechanism that occur in ductile materials during high rate deformation. The basic mechanism, which is now well known, is as follows. Plastic working is converted into thermal energy that heats up the material. Since most metals suffer a loss of strength as temperature increases, there is competition between work hardening and strain rate hardening on the one hand, and thermal softening on the other. At some critical condition softening wins, and a short time later localization into a shear band occurs with a drastic loss of shear strength. In some circumstances many small bands may form throughout a volume of material, in which case a general weakening occurs with the possibility of multiple failures and a general fragmentation. In other circumstances one band may dominate, and then material failure is restricted to just that one location.

In any case the designer would benefit from understanding the circumstances and processes by which localization occurs, for with understanding comes the possibility of control. There are important military applications in penetration mechanics, shock loading and impact. There are also important civil applications in crash worthiness and design of impact tools, but most important of all, in high speed metal working and forming, i.e. in fundamental manufacturing processes.

In all the applications mentioned above it should be useful to have available simple formulas that characterize major features of the deformation patterns, such as timing, intensity, morphology and location. If those formulas depend only on well-established physical properties, or at most on response functions that may be readily measured in any well-equipped physical laboratory, then they attain the highest level of usefulness and a status that merits the name *scaling law*.

Such laws may come from experimentation, but in the case of adiabatic shear bands the phenomena are sufficiently complicated and there are such a large number of non-dimensional parameters that mathematical analysis, sometimes guided by numerical analysis, is required to isolate the desired parametric effects. Since shear bands are very narrow in comparison with their other dimensions, one-dimensional simple shearing is a canonical problem that captures much of the observed phenomena and often shows how the physical and geometric parameters are organized. The full problem is highly non-linear and only solutions to the non-linear equations can show the full detail of the phenomena, but solutions to linearized equations also may show how the variables are organized and reveal major aspects of patterns that persist in the non-linear solutions.

2. MODEL EQUATIONS

In one dimension a simple set of equations that can serve as a model for plastic deformation in simple shear is as follows:

$$\begin{aligned} &\text{momentum:} && \rho v_t = s_y \\ &\text{energy:} && \rho c\theta_t = k\theta_{yy} + s v_y \\ &\text{flow law:} && s = F(\kappa, \theta, v_y) \\ &\text{work hardening:} && \kappa_t = M(\kappa, \theta, v_y). \end{aligned} \tag{1}$$

In these equations v is particle velocity parallel to the band, which extends in the x direction, s is shear stress, θ is temperature relative to a reference temperature T_0 and κ is the work hardening parameter. The density is ρ, heat capacity is c and thermal conductivity is k. Subscripts denote partial differentiation with respect to time, t, or the spatial coordinate, y. Equations (1) represent a rigid/plastic material with work hardening ($\partial F/\partial \kappa > 0$ and $M \geqslant 0$), rate hardening ($\partial F/\partial v_y > 0$) and thermal softening ($\partial F/\partial \theta < 0$). For a perfectly plastic material $M \equiv 0$.

A characteristic length is λ, a characteristic strain rate is $\dot{\gamma}_0$, a characteristic stress is given by $S_0 = F(\kappa_0, 0, \dot{\gamma}_0)$, a characteristic work hardening parameter is κ_0 and the initial absolute temperature is T_0 so that the initial relative temperature is zero. Two other physical parameters that will be needed are the coefficient of thermal softening $a = -F_\theta/F$ and the strain rate sensitivity $m = (\partial \log F)/(\partial \log \dot{\gamma})$.

3. TYPICAL RESULTS

For a rigid/perfectly plastic material (no work hardening) with linear thermal softening and power law rate hardening, a representative flow law may be written as

$$s = \kappa_0(1 - a\theta)(1 + bv_y)^m. \tag{2}$$

Since the strain rates of interest are high, eqn (2) may be rewritten as

$$s = K(1 - a\theta)v_y^m, \tag{3}$$

where $K = \kappa_0 b^m$. K may be thought of as the flow stress at a strain rate of 1 s^{-1}, although it has the rather unusual dimensions of stress–(time)^m.

Wright and Walter (1987) examined the response to a small perturbation in temperature for such a material, and in a typical calculation it was found that the perturbation grew slowly at first, with the temperature and strain rate just compensating each other so that the stress decreased slowly as if there were no perturbation at all. Then when a critical strain was reached, the stress dropped sharply while the temperature and strain rate increased extremely rapidly in the center of the perturbation as the localization formed. Finally, in a fully formed band the velocity profile showed a smooth but rapid, jump from one nearly constant value to another with an extremely rapid transition through a narrow zone. At the same time, in the fully formed band there was a further slow decrease of stress, a slow increase of the central temperature and no further change of temperature at a distance from the band since there was no further plastic work there. The overall picture then is one of nearly uniform velocity gradient changing rapidly to one where two nearly rigid blocks of material translate relative to each other with one sliding over the other, and with only a narrow transition zone separating the two.

When the fully developed band "pops in", the central strain rate may reach several orders of magnitude over the ambient rate, i.e. $\dot{\gamma}_{max}/\dot{\gamma}_0 = \mathbf{O}$ (10^2) or O (10^3), and the morphology of the fully developed band changes only slowly thereafter. It was found by numerical experiment (Wright and Walter, 1987), that the critical strain in a particular case depended on the nominal strain rate and that the dependence could be described by a U-shaped curve, as shown in Fig. 1. As the nominal strain rate decreased towards a small but finite value, the critical strain tended towards infinity, indicating complete stability, and as the nominal strain rate became large, the critical strain first decreased to a minimum and then increased towards infinity, again indicating stability at extremely high strain rates. In the left-hand branch at the lower strain rates, stability is achieved through heat conduction, and in the right-hand branch stability is achieved through inertia at high strain rates. The properties of the U-curve have been described in several papers, and several scaling laws have been obtained for the simple model with linear softening. For example, as shown by Wright (1990), the U-curve itself scales like

$$\frac{aS_0}{\rho cm}\Delta\gamma = U\left(\frac{\rho c\lambda^2\dot{\gamma}_0}{k}\cdot a\delta\theta\right) \geqslant C\log\frac{2m}{a\delta\theta}, \tag{4}$$

where $\Delta\gamma$ is the critical strain at localization and $\delta\theta$ is the magnitude of the perturbation in temperature. In eqn (4), note that the argument of U is proportional to $\dot{\gamma}_0$, which was the computational parameter in Wright and Walter (1987) that generated the U-curve in the first place.

The minimum in the U-curve can be found by examining the early growth rate of perturbations, as shown by Ockendon and Wright (1993). There it was shown that if the length scale is held fixed, as it would be in a Hopkinson torsion bar test, then the strain rate for maximum rate of growth, and hence the minimum in the U-curve, is given approximately by

$$\dot{\gamma}_0 = \frac{\pi^{4/3}m}{2^{1/3}}\left[\frac{kc}{\lambda^4 a^2 S_0}\right]^{1/3} + \frac{\pi^{2/3}m^2}{2^{2/3}\cdot 3}\left[\frac{S_0 c^2}{ka\lambda^2}\right]^{1/3}, \tag{5}$$

and this predicted well the minimum of the U-curve in Wright and Walter (1987). On the other hand, for fixed strain rate in an infinite medium, Ockendon and Wright (1993) showed that the wavelength that grows the fastest is given approximately by

$$L = 2\pi\left[\frac{m^3 kc}{\dot{\gamma}_0^3 a^2 S_0}\right]^{1/4} = 2\pi\left[\frac{m^3 kc}{\dot{\gamma}_0^{3+m} a^2 K}\right]^{1/4}. \tag{6}$$

Equation (6) may also be interpreted as the most probable minimum spacing for shear

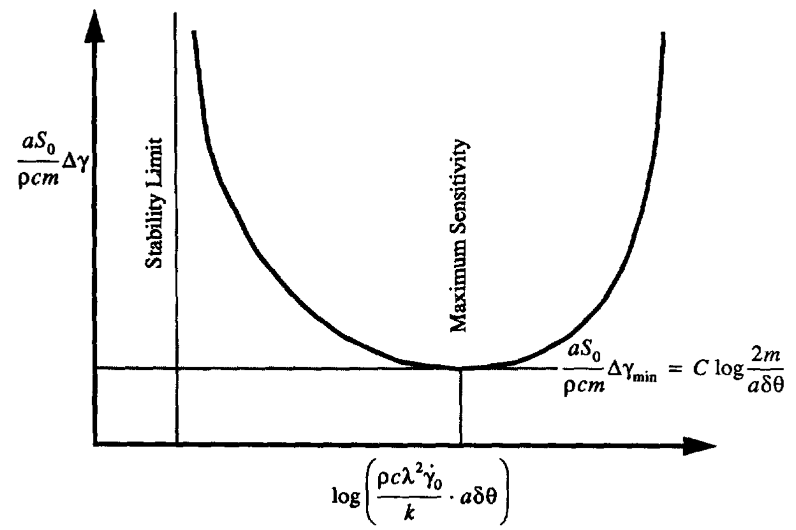

Fig. 1. Sketch of a typical U-curve.

bands in the region, and its reciprocal is the expected number density, N per unit length. With the nominal strain rate given and the band spacing now known, the velocity jump across one band must be given by

$$\Delta v = L\dot{\gamma}_0 = 2\pi\left[\frac{m^3 kc\dot{\gamma}_0^{1-m}}{a^2 K}\right]^{1/4}. \tag{7}$$

The cross-section of a fully formed shear band has a well-defined morphology, which may be calculated as the characteristic function in a non-linear eigenvalue problem, as shown by Wright and Ockendon (1992) and extended by Glimm *et al.* (1993). It turns out that the maximum strain rate in the band, $\dot{\gamma}_{max}$, the half-width of the most rapidly shearing section of the band, δ, and the velocity jump across the band, Δv, are related by $\delta\dot{\gamma}_{max} = \Delta v/2$, as well as by the following formulas:

$$\dot{\gamma}_{max} = \left[\frac{1-m}{m}\cdot\frac{aK}{k}\right]^{\frac{1}{1-m}}\cdot\left(\frac{\Delta v}{2}\right)^{\frac{2}{1-m}} = \left[\frac{\pi^4(1-m)^2 mcK}{k}\cdot\dot{\gamma}_0^{1-m}\right]^{\frac{1}{2(1-m)}}$$

$$\delta = \left[\frac{m}{1-m}\cdot\frac{k}{aK}\right]^{\frac{1}{1-m}}\cdot\left(\frac{\Delta v}{2}\right)^{\frac{1+m}{1-m}}$$

$$= \pi^{-\frac{1+m}{1-m}}(1-m)^{-\frac{1}{1-m}} m^{\frac{1-3m}{4(1-m)}} k^{\frac{3-m}{4(1-m)}} a^{-1/2} K^{-\frac{3-m}{4(1-m)}} c^{-\frac{1+m}{4(1-m)}} \dot{\gamma}_0^{-\frac{1+m}{4}}. \tag{8}$$

The first form for each of $\dot{\gamma}_{max}$ and δ applies to the jump across a single band, which may be forced by the experimental setup, and was given by Wright and Ockendon (1992). The second form applies when multiple bands can form freely in the bulk material and follows from use of eqn (7). Equations (6)–(8) may now be used to calculate, or at least estimate, all physical features associated with band formation in a region undergoing high strain rate, including an estimate of minimum fragment size at failure if size is assumed to be correlated with band spacing.

The percentage of material involved in localization is given by the ratio δ/L:

$$\frac{\delta}{L} = \frac{1}{2}\left[\frac{1}{\pi^4(1-m)^2}\cdot\frac{k\dot{\gamma}_0^{1-m}}{mcK}\right]^{\frac{1}{2(1-m)}}. \tag{9}$$

It is more difficult to obtain solid analytical results for a rigid/work hardening material because of the extra degree of freedom introduced by the evolution of the work hardening parameter. Nevertheless, some results have been obtained, at least for special cases. With power law work hardening, that is $\kappa = \kappa_0(1+\gamma/\gamma_0)^n$ in slow loading and $M = (n/\gamma_0)(\kappa/\kappa_0)^{-1/n} s v_y$ in high rate loading, it turns out that there is an equivalence between defects in temperature and defects in strength if they are properly scaled and their relative magnitudes are only a few percent (Wright, 1994). For this case, other than in an initial boundary layer in time, all aspects of the solution depend only on the combination

$$\delta\theta(y) - \frac{\gamma_0}{\rho c}\delta\kappa(y), \tag{10}$$

where eqn (10) refers only to the initial perturbations in temperature and strength. For the work hardening case, as for the perfectly plastic case, numerical examples have indicated that the critical strain at localization depends on the nominal strain rate through a U-curve in much the same manner as for the perfectly plastic case (Walter, 1992). The critical strain now is composed of two parts: first, the strain to maximum stress where strain softening begins; and second, the remaining strain to localization. It turns out that the first part may

Table 1. Effect of nominal strain rate on band characteristics

$\dot{\gamma}_0$ (s^{-1})	L (mm)	Δv (m s^{-1})	$\dot{\gamma}_{max}$ (s^{-1})	δ (μm)	L/δ
200	11.5	2.3	1.73×10^6	0.67	1.7×10^4
2000	2.0	4.0	5.47×10^6	0.37	5.5×10^3
20,000	0.36	7.1	17.3×10^6	0.21	1.7×10^3
200,000	0.063	12.5	54.7×10^6	0.11	5.5×10^2

† In this example $m = 0.02$, $a = 0.002$ K^{-1}, $\kappa_0 b^m = 5 \times 10^8$ Pa, $k = 50$ W m^{-1} K^{-1}, $c = 500$ J kg^{-1} K^{-1}.

be approximated from uniform deformation without regard to perturbations, and numerical examples given by Wright (1994) have suggested that the final part may often be approximated by the following expression,

$$\sqrt{\left(\frac{\pi m}{2n}\right)} \cdot \frac{aS_0\Delta\gamma}{\rho cm} \geqslant \log\frac{m}{a[\delta\theta-(\gamma_0/\rho c)\delta\kappa]}, \tag{11}$$

where $[\delta\theta-(\gamma_0/\rho c)\delta\kappa]$ is the maximum initial perturbation.

4. DISCUSSION

Although results are more complete for the perfectly plastic case than for the work hardening case, the similarity between eqns (4) and (11), which make predictions regarding the timing of localization, is very striking. The extra factor on the left-hand side of eqn (11) comes from the asymptotic evaluation of an integral at the peak stress, as explained in detail by Wright (1992). In addition, the total strain at localization must include the strain to peak stress which, of course, is zero for the perfectly plastic case. Both formulas show the necessity for proper scaling in order to understand the interplay of material properties and timing of localization.

It may be speculated that the various formulas, eqns (5)–(9), which were obtained for the perfectly plastic case, also have some bearing for the work hardening case. Except for numerical factors and other factors arising from rate dependence, these five formulas each have a counterpart for a rate-independent material, as described by Grady and Kipp (1987), and have the same essential dependence on k, c, a, K and $\dot{\gamma}_0$. Table 1 gives an example of the values predicted by these formulas for a material with properties like those of a typical steel. Note particularly how the spacing, and hence the number density, change with nominal strain rate roughly as $\dot{\gamma}_0^{-3/4}$ and $\dot{\gamma}_0^{3/4}$, respectively, but the maximum strain rate in a band changes only as $\dot{\gamma}_0^{1/2}$, and the width of a shear band changes more slowly still, varying only as $\dot{\gamma}_0^{-1/4}$. This accords with the observation that for a given material shear bands have a similar appearance, even though loading rates may vary widely. Note especially how the dependence for a single band, which varies essentially as $\dot{\gamma}_0^{-1}$, switches to a dependence on $\dot{\gamma}_0^{-1/4}$ due to the multiplication of bands being favored, rather than increased intensity in a single band.

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